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Quantifying the randomness of the forex market

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Abstract

Currency markets are international networks of participants opened all day during weekdays without a supervisory entity. The precise value of an exchange pair is determined by the decisions of the central banks and the behavior of the speculators, whose actions can be determined on the spot or be related to previous decisions. All those decisions affect the complexity and predictability of the system, which are quantitatively analyzed in this paper. For this purpose, we compare the randomness of the most traded currencies in the forex market using the Pincus Index. We extend the development of this methodology to include multidimensionality in the embedding dimension, to capture the influence of the past in current decisions and to analyze different frequencies within the data with a multiscale approach. We show that, in general, the forex market is more predictable using one hour ticks than using daily data for the six major pairs, and present evidence suggesting that the variance is easier to predict for longer time frames.

Keywords:
Complexity, Econophysics, Information Theory, Randomness

1. Introduction

The foreign exchange market (forex) is the largest market in the world and determines the exchange rates for every currency. The participants in the market set the relative value of each currency pair by buying or selling positions, and these actions are influenced by personal beliefs, trends or public announcements of central banks.

Unlike stock markets, forex is not being subject to an specific supervisory entity and is globally decentralized, open to banks, commercial companies and private agents. The price of a currency pair at a given time is supposed to be a reflection of economic factors, political conditions and the psychology of the participants.

The six most traded forex pairs analyzed in this paper are: EUR/USD (euro/US dollar); USD/JPY (US dollar/Japanese yen); GBP/USD (British pound sterling/US dollar); AUD/USD (Australian dollar/US dollar); USD/CAD (US dollar/Canadian dollar); USD/CNY (US dollar/Chinese renminbi).

The forex market has been the subject of intensive research for a long time, frequently focusing on the predictability of its values but also on its variance. In this paper, we analyze the number of patterns within the data for the returns and its variance in different timeframes, 1-hour (H1), 4-hours (H4) and daily.

Research on complexity includes a variety of algorithms and analysis techniques that usually come from the physical or mathematical realm [1]. Complex systems are entangled by nonlinearly interacting elements and are found in different fields such as the brain or financial markets. A review on the meaning of complexity and detailed analyses in those and other processes can be found in [2].

When dealing with the currency market, its movements can be separated into high-frequency variations [3] and slower movements responsible for the trends [2]. This paper deals with the later, and explores its complexity relying on the concept of entropy as defined in Information Theory and Kolmogorov complexity.

In the field of Information Theory, entropy is a magnitude that quantifies the uncertainty of a measure. On the other hand, this paper follows the approach of Chaitin [4] and Kolmogorov [5] by defining complexity as in algorithmic information theory, which takes into account the order of the points in a sequence. In this view, a chain is random if its Kolmogorov complexity is at least equal to the length of the chain.

The benefit of the connection between information content and randomness is that it provides a way to quantify the complexity of a dataset without relying on models or hypothesis about the process generating the data. By comparing the entropy of our system with the maximum entropy rate possible, we can determine the degree of randomness of a series; a complex (total random) process is defined as that process lacking pattern repetition.

The use of the entropy rate to study the complexity of a time series is not limited to stochastic processes. Sinai [6] introduced the concept of entropy to describe the structural similarity between different dynamic systems that preserve the measurements, giving a generalization of Shannon entropy for dynamic systems, known as Kolmogorov-Sinai entropy (KS). Unfortunately, KS entropy is sometimes undefined for limited and noisy measurements of a signal represented in a data series.

To overcome that limitation, Grassberger and Procaccia [7]

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used the Renyi entropy to define the correlation integral, which in turn was used by Eckmann and Ruelle [8] to define the ϕ functions as a conditional probability. This ER entropy is an exact estimation of the entropy of the system. Building upon those ϕ functions, Pincus [9] described the methodology of ApEn, useful for limited and noisy data, providing a hierarchy of randomness based on the different patterns and their repetitions.

ApEn measures the logarithmic probability that nearby pattern runs remain close in the next incremental comparison: low ApEn values reflect that the system is very persistent, repetitive and predictive, with apparent patterns that repeat themselves throughout of the series, while high values means complexity in the sense of independence between the data and a low number of repeated patterns. The readers are encouraged to read a recent comprehensive tutorial on these algorithms [10].

To use Approximate Entropy, it is necessary to specify two parameters, the embedding dimension (m) and the tolerance of the measure (r), determined as a percentage of the standard deviation. Once the calculations have been performed, the result of the algorithm is a positive real number, with higher values indicating more randomness. However, those values are dependent on the characteristics of the dataset such as the influence of the past in the future prices or the volatility of the prices.

In order to obtain a measure of randomness suitable for comparisons between evolving datasets, the Pincus Index (PI) was introduced as a measure of the distance between a dataset and the maximum possible randomness of that system [11]. A value of PI equal to zero implies a totally ordered and completely predictable system, whereas a value equal to or greater than one implies total randomness and unpredictability. The added benefit of the Pincus Index is that, unlike ApEn, it is suitable for comparisons between different markets. This paper completes the development of that index by introducing different kinds of multidimensionality in the measure. Thus, knowledge of the PI would be useful to fully understand the concepts here presented and how the several levels of complexity of this measure are captured.

The Pincus Index was designed [11] to be independent on the parameter r by choosing the maximum value of Approximate Entropy (MaxApEn), but the index is still dependent on the selection of the embedding dimension (m). This parameter is related to the memory of the system and accounts for the length of the patterns compared in the sequence. Techniques to determine the optimum value of the embedding dimension include the use the mutual information and false nearest neighbour method [12][13][14], but since different markets may have different embedding dimensions, the comparisons with a fixed m could be biased. To account for that possibility, we follow Bolea and coauthors [15] in the definition of a Multidimensional index. Since such an index was based on MaxApEn, its extrapolation to a Multidimensional Pincus Index is straightforward and provides a parameter-free index which allows for comparisons between evolving systems.

Besides multidimensionality in embedding dimension, dynamic systems may be composed of processes at different frequencies with correlations at multiple time scales. Therefore, in the characterization of complexity, the comparison of different frequencies may lead to incorrect conclusions. Costa and coauthors [16] proposed a multiscale procedure to capture those correlations, showing its efficiency in distinguishing complexities in different dynamical regimes. To describe the complexity of a time series at different levels, Costa and coauthors [17] generalized the multiscale procedure to consider the complexity of higher statistical moments of time series. Here, we extend that methodology to create a new Multiscale Pincus Index, showing how it is useful to correctly quantify the complexity of trading in different timeframes and different statistical moments.

Methods and results

On the calculation of the Pincus Index

The Pincus Index (PI) captures the distance from a situation of total randomness for a given dataset, measured against shuffled versions of the same data. To better quantify complexity and provide an index that is independent of the tolerance r, it is constructed based on the maximum value of Approximate Entropy (MaxApEn). The steps to compute the PI include the determination of the MaxApEn of the original sequence and the MaxApEn of bootstrapped versions. Then, we use the median value (50% percentile) of the empirical distribution of the bootstrapped versions to calculate the value of the Pincus Index, and the 5% and 95% percentiles of the empirical cumulative distribution function to calculate the extremes of the index. The rationale is simple: if the degree of randomness of the original sequence is similar to the shuffled versions, the PI will be close to one, indicating randomness. If, on the other hand, the original sequence is ordered, the PI will capture the distance from randomness as a fraction. For a detailed explanation of the methodology and several examples of application, the reader is encouraged to see [10][11].

The Pincus Index is based on Approximate Entropy. When the number of data (N) is large, ApEn can be approximated by Equation 1. The error committed in this approximation is estimated to be smaller than 0.05 for $N-m+1 > 90$ and smaller than 0.02 for $N-m+1 > 283$ [18].

$$ApEn(m,r,N) \simeq \\ -\frac{1}{N-m}\sum_{i=1}^{N-m}\log\frac{\sum_{j=1}^{N-m}[\text{times that } d[|x_{m+1}(j)-x_{m+1}(i)|] < r]}{\sum_{j=1}^{N-m}[\text{times that } d[|x_m(j)-x_m(i)|] < r]}. \tag{1}$$

where m is the length of the vectors being compared, and d measures the scalar distance between the vectors in a component-wise way.

The Sample Entropy (SampEn) algorithm has been designed to avoid the self-bias included in ApEn [18], which is mathematically formulated as [10]:

$$SampEn(m,r,N) = -\log \frac{\sum_{i=1}^{N-m}\sum_{j=1,j\neq i}^{N-m}[\text{times that } d[|x_{m+1}(j) - x_{m+1}(i)|] < r]}{\sum_{i=1}^{N-m}\sum_{j=1,j\neq i}^{N-m}[\text{times that } d[|x_m(j) - x_m(i)|] < r]} \quad (2)$$

It is often said that SampEn is largely independent on the number of points because, unlike ApEn, it does not include a prefactor $\frac{1}{N-m}$. However, it must be noticed that such independence is only true for homogeneous series, and it does not hold for general situations [19]. In general, randomness depends for both algorithms on m and N [20].

In Figure 1 we show the different behavior of ApEn and SampEn for 50 pseudo-random binary (left) and decimal (right) chains using $m = 2$; we use $r < 1$ to make the analysis independent of this parameter, given their well-defined alphabet. As it can be seen, the mean value of SampEn reaches the asymptotic limit of $\log k$ faster but with a larger standard deviation than ApEn.

In the construction of the Pincus Index, we calculate the ratio $\text{MaxApEn}_{original}/\text{MaxApEn}_{shuffled}$. Since those quantities are calculated using the same values of m and N, the ratio between them does not include the prefactor $\frac{1}{N-m}$ appearing in ApEn in Equation 1. This fact makes the PI independent of the number of points in the same way that SampEn (i.e., for white noise, or a homogeneously generated sequence, it captures the randomness independently of N).

Another reason for the construction of SampEn was the self-counting introduced in the calculation of ApEn: note that the definition of SampEn explicitly avoids that situation by limiting $j \neq i$ in Equation 2. That bias can be as high as 20% or 30% if the number of points is low [18]. In this regard, since the PI is constructed as a ratio and the bias in ApEn is present in both the nominator and the denominator, the overall bias is modulated and severely corrected, providing a better measure of complexity. It should be emphasized that the PI does not measure randomness specifically but how far away a series is from total randomness.

The threshold: r_{max} and MaxApEn

The use of an incorrect parameter selection when using ApEn or SampEn can lead to inaccurate estimations of the complexity of datasets. By means of MaxApEn we can prevent the arbitrary selection of the threshold of r, which changes depending on the complexity of the sequence.

Restrepo et al. [21] showed that the combined use of MaxApEn and r_{max} can help to correctly characterize the complexity. Using a dataset containing daily values of EURUSD from 2006 to 2010, we show in Figure 2 that r_{max} changes with the embedding dimension selected. The distance between the maximum value of the threshold for the original (red line) and the shuffled series (black line) shows that using a fixed common valued for the threshold would lead to misleading results. Thus, even though the recommended range for r is commonly $[0.1\sigma, 0.25\sigma]$, that region does not guarantee to capture the complexity correctly for all the values of the embedding dimension. It is advised to use a value equal to or greater than the value of r_{max} [10] to assure the relative consistency; the comparison with different values of r beyond the maximum would lead to the same qualitative characterization of the order of the system.

As explained in [21], the differences in r_{max} for the original and shuffled versions can be used as a mean to discern between systems in noisy datasets with low number of samples N. Albeit in this work we focus on the development of the Pincus Index, we take the opportunity to recall that, for some dynamical regimes, these combined techniques could provide a better characterization of the systems, and show that the recommended range may not be adequate depending on the embedding dimension m.

The embedding dimension: multidimensional analysis

In the methodology to calculate the Pincus Index, the tolerance r is automatically selected as the value which maximizes Approximate Entropy [11]. However, the selection of the embedding dimension is a requirement for the calculations. The embedding dimension determines the length of the patterns being compared, and it is related to how much information from the past is used to determine the future values. In the search of a parameter-free application of Approximate Entropy, Bolea et al. [15] proposed the use of MaxApEn combined with a multidimensional analysis by adding the contribution of MaxApEn over a wide range of embedding dimensions to capture the influence of previous values.

Since a priori the memory of the system is unknown and it may change in evolving datasets like the forex markets, adopting the same methodology as Bolea and coauthors, it is straight forward to build a Multidimensional Pincus Index (MPI) independent of both r and m, by defining:

$$MPI = \frac{\sum_{m_i=1}^{m_{max}} MaxApEn_{original}(m_i)}{\sum_{m=1}^{m_{max}} MaxApEn_{shuffled}(m_i)} \quad (3)$$

We illustrate the behavior of this new multidimensional index in Figure 3 using the EURUSD exchange rate as an example. Figure 3 (left) shows the MaxApEn for different embedding dimensions for the original series (black line) and pseudo-randomized versions of the same data (red) for a dataset containing daily values of EURUSD from 2006 to 2010. Based on those values, we show how the MPI changes when we consider only the previous m values using Equation 3. We observe that, by adding the contribution of larger embedding dimensions, the MPI varies to capture the increased information in the complexity of the series. The right side of Figure 3 shows the MPI accounting for the contribution of the embedding dimension up to 15 ($\text{MPI}(m_{max} = 15)$) for rolling windows of four years of EURUSD daily exchange rate, i.e., approximately $N \sim 1000$ points.

The rationale for the inclusion of multidimensionality is its ability to capture complexity in a greater extent, as shown by Bolea et al. [15]. Specifically for the forex or stock markets, or when drawing comparisons between different systems, it is

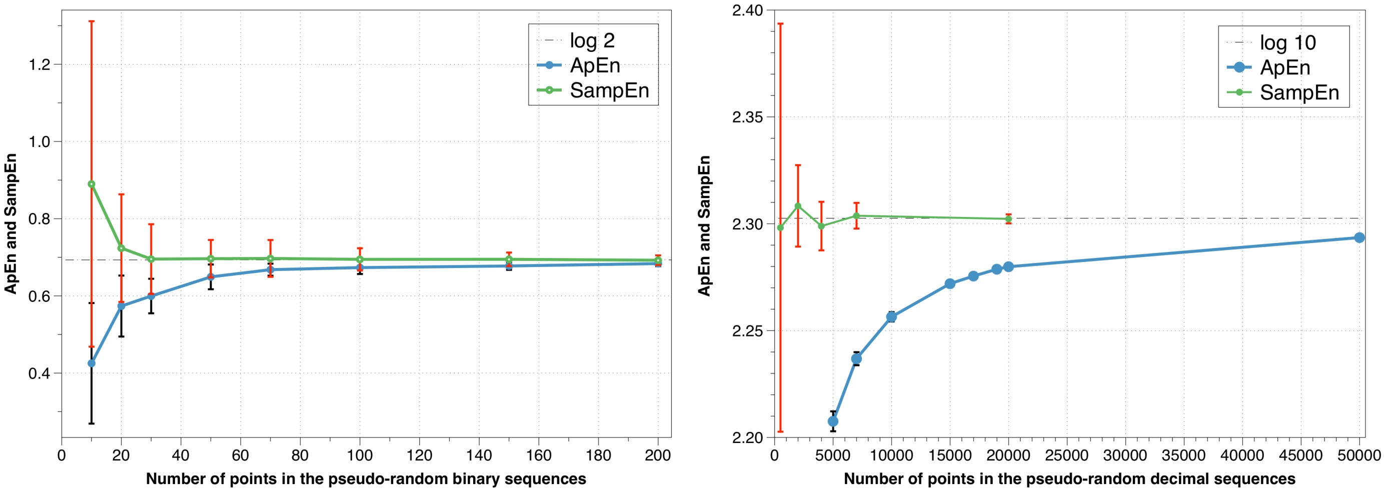

Figure 1: ApEn, SampEn, and asymptotic lines depending on the alphabet for 50 pseudo-random binary (left) and decimal (right) sequences.

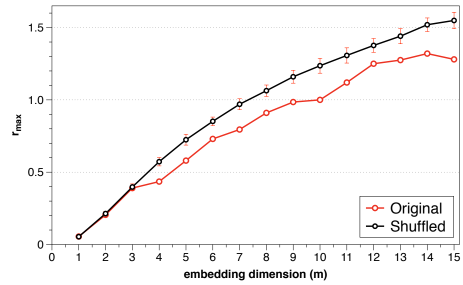

Figure 2: Value of r for which MaxApEn is reached for the original series (red line) and the average of one hundred shuffled versions (black line). The average of the standard deviations (bars) for the shuffled versions is 0.038 and increases with the embedding dimension.

not guaranteed that the optimal value of the embedding dimension would be the same. In general, randomness depends on m and N, as shown by Pincus and coauthors when they defined the maximum $\{m, N\}$-randomness [20][22]. The value of the Pincus Index is its aptness to make comparisons between systems by measuring the distance of each series against the maximum randomness of each alphabet. We shall remember that both ApEn and SampEn provide relative values, and may be unsuitable for comparisons. By including multidimensionality in the definition of the MPI, we obtain an index independent of preselected parameter values for both r and m which can be used with evolving datasets.

Sampling frequency: multiscale entropy

Another variable must be taken into account in order to capture complexity in all of its forms, which is the different frequencies within the data. It is not uncommon for dynamical systems to be composed of subprocesses emerging at different time scales. That situation is often observed in the markets when the trend at a certain frequency domain, let us say 15 minutes, is not the same (or even the complete opposite) as the trend at 1 day data.

To account for that possibility, Costa et al. [16] designed the multiscale entropy (MSE) procedure based on the approach proposed by Zang [23][24]. This measure is based on a weighted sum of scale dependent entropies, and it has been used extensively since its appearance in the literature [25]. The main idea is the construction of coarse-grained time series determined by a certain scale factor τ, averaging different time scales from the original time series. The coarse-graining procedure reduces the length of the sequence by a scale factor τ, obtaining a coarse-grained time of length N/τ, with N the original length. Thus, the larger the scale factor used, the shorter the resulting length of the coarse-grained time series.

This procedure has become a prevailing method to quantify the complexity of data series and it has been vastly applied in many different research fields, including finances [26]. After the creation of the coarse-grained sequences, the entropy of each sequence is calculated and added up to obtain a multiscale entropy value. More detailed instructions of the methodology can be found in Costa and coauthors works [16][27].

The MSE methodology has generally been applied in conjunction to Sample Entropy, given the above-mentioned fact that is less dependent on the time series length since it does not include a prefactor in Equation 2. However, similarly to the comparison of different time series, different time scales may have different alphabets and the comparisons using the same parameters may be biased. Some traded time frames will show higher variability, while the variations at different frequencies may show lower changes and averaged values. Furthermore, Sample Entropy uses a fixed value of the tolerance filter r which may not be adequate for all frequencies. This hinders the applicability of Sample Entropy to characterize the randomness level appropriately.

As seen in the previous sections, the value of r which captures the maximum complexity is different for each sequence; since the Pincus Index is based on MaxApEn, which automatically adapts to the maximum complexity of each frequency, this index is able to capture the distance from total randomness of the different frequencies. As an example, Figure 4 shows the

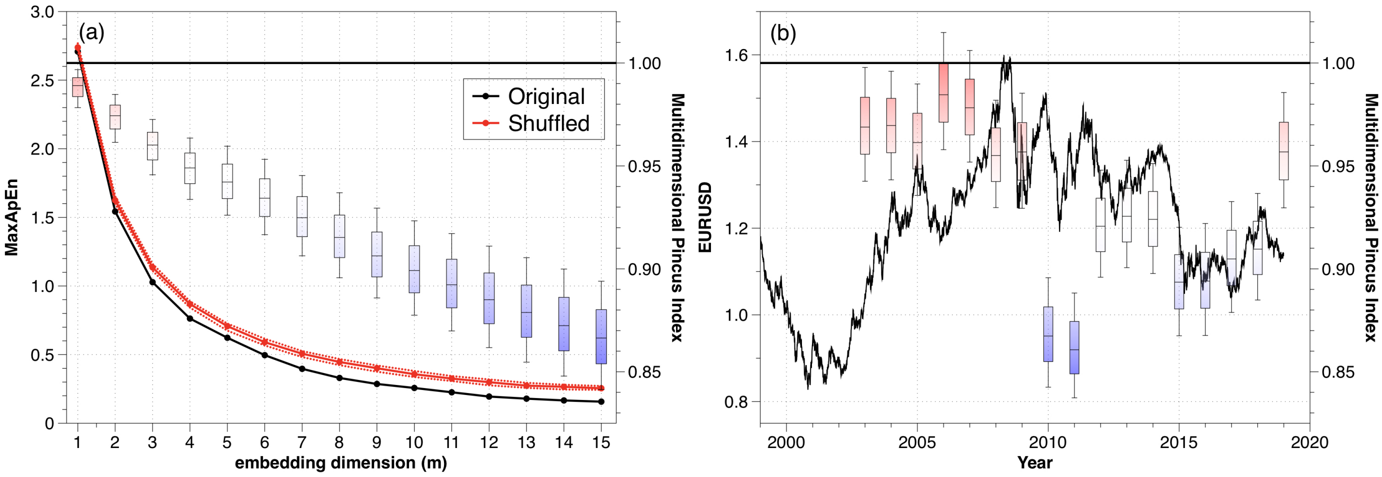

Figure 3: Multidimensional Pincus Index

results of the Pincus Index for values traded Daily, at four hours (H4) and at one hour (H1) frequencies for $m = 2$. We present the results for the six major traded pairs to display the evolution of the different frequencies.

In a previous communication we showed the effect of including more or less number of points (see Supplementary Information in [11]). In this paper, our interest lies on characterizing the different frequencies knowing that approximately the same number of points are used to draw comparisons about their complexity. To that end, we use rolling windows of approximately $N \sim 1000$ for all frequencies showed in Figure 4, corresponding to four years in daily values, eight months in H4 data, and two months in H1 data. Importantly, it can be seen in the figure that, in general, one hour frequencies are less random than daily values, but the specific values of the Pincus Index change with the epoch and the frequency for each of the six considered markets. It is also noticeable that the shape of the PI for the EURUSD daily values presented in Fig. 4(a) is the same as the shape showed for the multidimensional PI analysis in Figure 3, but with higher values. As far as we have computed, adding the information contained in higher embedding dimensions makes the sequence less random, but does not change the overall shape of the analysis between different rolling windows.

As in previous observations when dealing with the stock markets [11], we find that the degree of predictability increases in times of crisis or sharp changes in the values; in those moments, the market chains falling sessions creating more patterns repeated by the agents. Previous studies with high-frequency data have studied the changes during these kinds of events in the short time reaction of the markets [3], and in our analysis, we find examples of that behavior in Fig. 4(c) and (d). At those moments, the GBP and AUD pairs concatenated several months of depreciation with respect to the USD, making ApEn (the numerator of the Pincus Index) to decrease accordingly.

Higher statistical moments: Generalized Pincus Index

Finally, in this section we study a different type of dimensionality, this time by considering the statistical moments. In the MSE procedure, the coarse-graining of the original series consists on the averaging of the time series, using the first moment (mean). However, it is well known that, for time series in general and for financial time series in particular, higher moments such as the variance, skewness or kurtosis contain valuable information different from the mean, which can help to select τ.

Aware of this fact, Costa and coauthors extended their methodology to a Generalized Multiscale Entropy [17], MSEn, being n the order of the moment used in the calculations. In this generalized methodology, different moments are used in the coarse-graining procedure, helping to characterize the time series based on information not contained in the first moment. Hence, when the MSE is applied to higher moments such as skewness and kurtosis in financial time series, the MSEn is more effective to capture changes in the dynamics, providing valuable information [28]. Furthermore, some authors have proposed the use of a Refined Generalized Multiscale entropy [29].

In Figure 5, the PI is calculated for $m = 2$ for different coarse-graining values τ, with $\tau = 1$ corresponding to hourly data and $\tau = 24$ to daily values. We can obtain useful information from this figure which helps us to understand the dynamics within the data. First, by looking at Figure 5 (left) we observe that the PI is low four hourly ($\tau = 1$) and daily data ($\tau = 24$), while other less traded frequencies in between such as $\tau = 7$, 11, 18 or 21, have higher values of PI indicating more randomness. It is important to notice that MaxApEn or SampEn alone are unable to capture those features because, as stated before, both are dependent on the number of points considered for non-homgeneous systems. In this example, the sequence with $\tau = 1$ consists of 12309 points while the last one with $\tau = 24$ has only 512. On the contrary, as the Pincus Index is computed for each sequence, the behavior of this index reflects the dynamics of the different frequencies. Furthermore, the choice of r for SampEn is rather arbitrary as evidenced in Figure 5 (right); we can observe that the value for which ApEn reaches its maximum changes drastically with the coarse-graining and thus keeping a

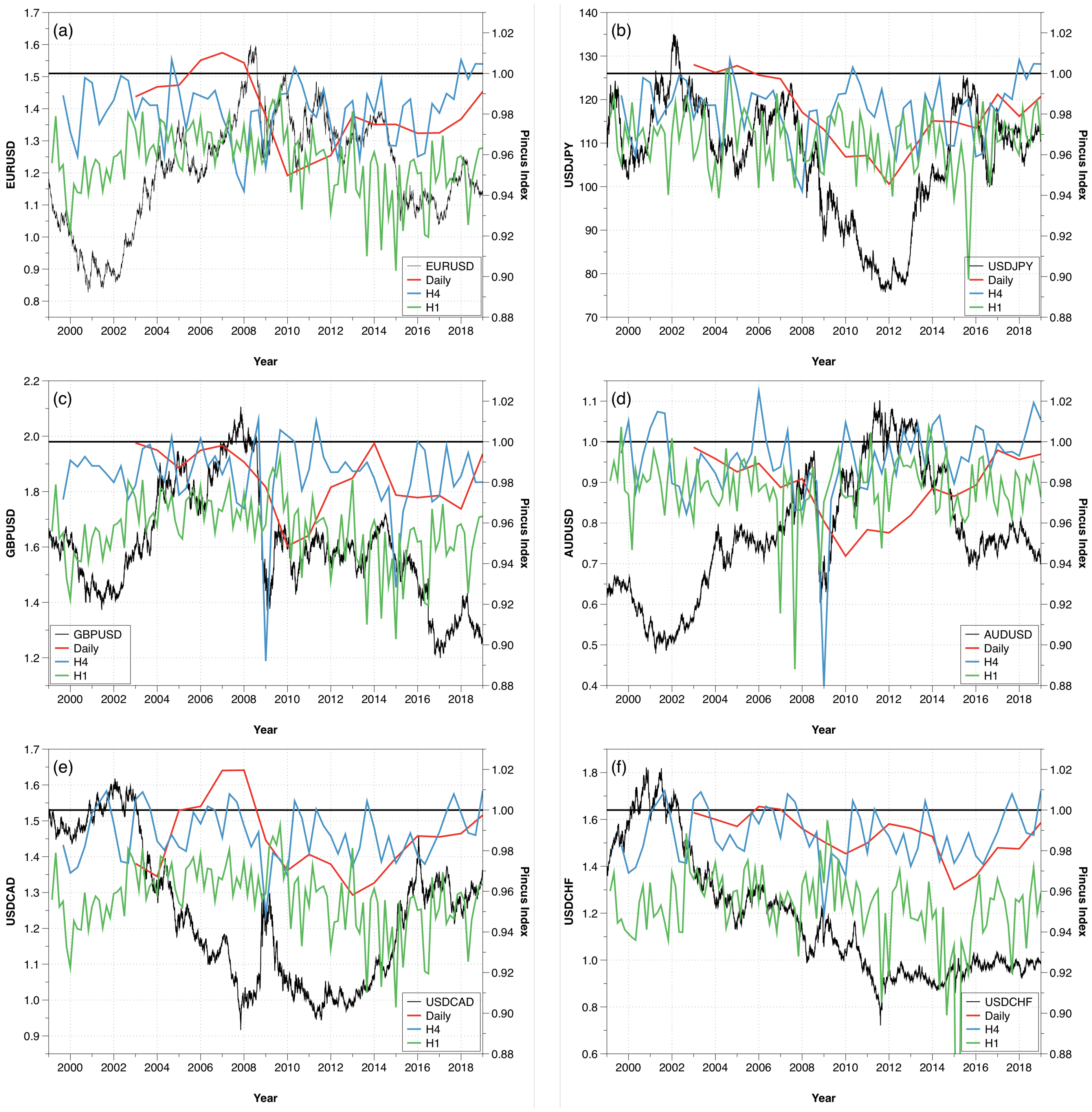

Figure 4: Pincus Index with m=2 for Daily, H4 and H1 data for the six major forex pairs.

fixed value of r for SampEn does not guarantee that the complexity is accurately captured. We have used the same values as Costa et al. for r in the SampEn calculations. The difference in the choice of r in the first and second moment analyses is because the amplitudes of the variance coarse-grained time series are much smaller than those of the mean coarse-grained time series [17].

In the comparison of financial time series, measures such as the volatility of the returns have great relevance [30][31]. The Pincus Index can also be applied to characterize the complexity of those higher moments by using the same generalization mechanism. Figure 6 shows the Pincus Index of the second moment, the variance, for the log-ratio series of EURUSD hourly data from 2008 to 2010 using $m = 2$. In Figure 6 we see that the variance is more predictable for daily data ($\tau = 24$) than for hourly data and, in general, is more predictable than the logarithmic return of the price itself showed in Figure 5, in agreement with recent research [32].

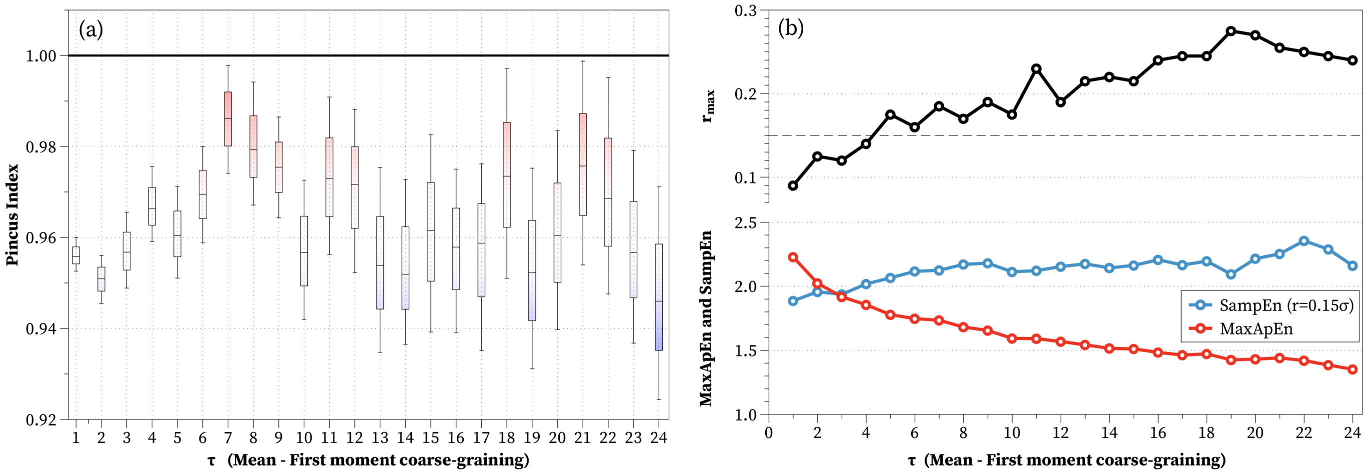

Figure 5: Left: Multiscale PI for the first moment ($m = 2$). Right: Corresponding r_{max}, MaxApEn and SampEn($r = 0.15\sigma$) for $m = 2$ for different τ using the EURUSD logratio series for H1 from 2008 to 2010.

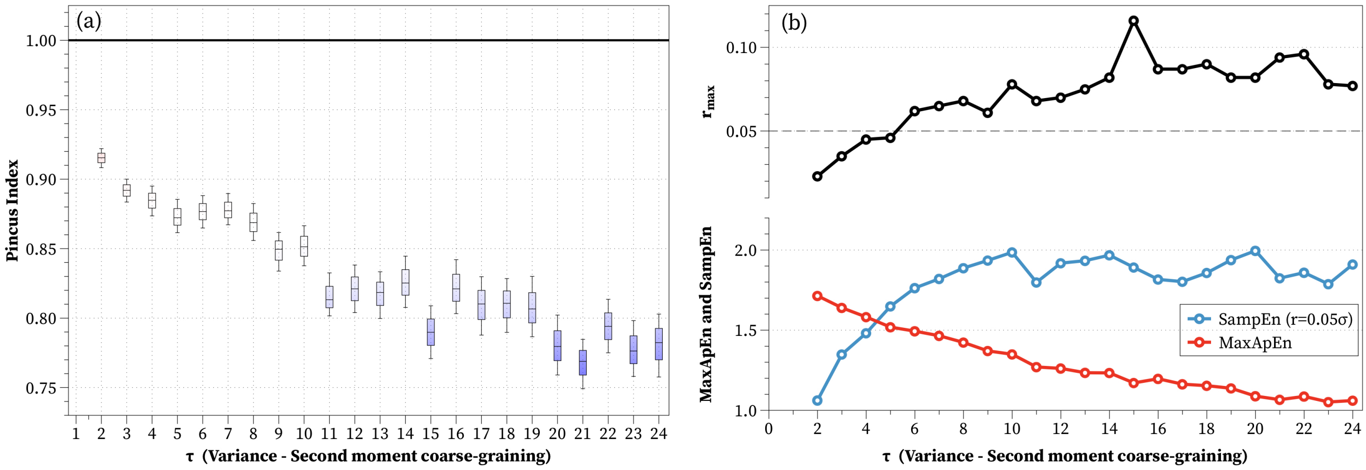

Figure 6: Left: Multiscale PI for the second moment ($m = 2$). Right: Corresponding r_{max}, MaxApEn and SampEn($r = 0.05\sigma$) for $m = 2$ for different τ using the EURUSD logratio series for H1 from 2008 to 2010.

Usually, the MSE devised by Costa and coauthors is interpreted as follows. If the MSE curve shows a monotonically decreasing trend, it is considered that only the smallest scale, i.e. the first values of τ contain information [29]. Similarly, if the trend is monotonically increasing only the larger scales, i.e. large values of τ, contain information. In our analysis, we can see the MSE curves of SampEn in the right side of Figures 5 and 6. We see the increasing trend in both cases, which would be interpreted saying that the complexity of larger values of τ is higher. However, when compared with the left side of those figures, we see that the Multiscale Pincus Index permits, not only to characterize the complexity of each frequency, but also to make comparisons between them.

The original formulation of MSE adds the entropy values of each coarse-grained series for a selected range of scales $\tau = 1, \ldots, \tau_{max}$, making the result dependent on the selected range. Wu et al. [33] proposed that the result would be better defined as the mean of the τ entropy values, suggesting the name Composite Multiscale Entropy (CMSE) for it. If one were to use the Pincus Index for a multiscale analysis, it would be desirable to continue normalizing the system to obtain a comparable metric by dividing the sum by the number of partitions. This leads to the definition of a Composite Multi Scale Pincus Index:

$$CMSPI = \frac{\sum_{\tau_i=1}^{\tau_{max}} PI(\tau_i)}{\tau_{max}} \tag{4}$$

Discussion and conclusions

The objective of this paper was to continue with the development of the Pincus Index to capture the different dimensions of the complexity of a dataset, and to use the methodology to analyze the six major traded pairs of the forex market.

We have shown that the methodology of the Pincus Index can be easily extended to capture the information contained in multiple embedding dimensions. Since different markets may have different optimal values for this parameter, the comparison of the MPI is useful to discern the complexity of the systems, as long as the maximum embedding dimension used in the analysis is the same for the markets considered.

Another source of complexity is the behavior of the system at different modes, and we have shown that the Pincus Index captures the complexity depending on the frequency. The main results of this work are the suggestions that the six analyzed markets are more predictable when using hourly data for a fixed number of points, and that less traded frequencies are more random than usually traded frequencies such as one hour or one day. Finally, as an example of generalization, we have analyzed higher moments to study the predictability of the variance, showing that the variance of daily data is more predictable than that of lower frequencies.

The analysis of the complexity of a dataset consists of multiple dimensions. In a complete characterization of the complexity of a time series, a multiscale approach using the Pincus Index can be developed in conjunction to a multidimensional embedding dimension analysis to fully account for the dynamics at different frequencies. However, given the computer power required to perform those calculations, instead of focusing on one market in particular we have preferred to show the behavior of the six major traded pairs.

The methodology presented in this paper is not restricted to the forex market or the field of economics in any way, and can be used to study any dataset. In particular, ApEn and SampEn have their roots in physiological studies and we believe that the presented methodology will be useful in that field. Source codes in R programming language for the determination of ApEn and SampEn are available in [10].

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